Functional Callan-Symanzik equation

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Abstract

We describe a functional method to obtain the exact evolution equation of the effective action with a parameter of the bare theory. When this parameter happens to be the bare mass of the scalar field, we find a functional generalization of the Callan-Symanzik equations. Another possibility is when this parameter is the Planck constant and controls the amplitude of the fluctuations. We show the similarity of these equations with the Wilsonian renormalization group flows and also recover the usual one loop effective action.

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I. INTRODUCTION

The renormalization group originally provides an insight into the scale dependence of the coupling constants [1]. Another, more recent use is to perform a partial resummation of the perturbation expansion by making an infinitesimal change of the cutoff in a time and using the functional formalism [2]- [7]. Both goals are realized by the blocking procedure, the successive elimination of the degrees of freedom which lie above the running ultraviolet cutoff. The resulting evolution equation yields the dependence of the coupling constants on the cutoff. This procedure suffers from consistency problems with the gradient expansion if we consider a sharp cutoff. To avoid it one usually introduces a smooth cutoff, but its form is not unique and the physical implications are not clear either. Moreover, in the framework of gauge theories, the slicing procedure in momentum space is not gauge invariant.

The "exact" renormalizaton program [3]- [7], with the setting up of the flow equations for the coupling constants, helps to compute the effective action of the theory in the limit where the cutoff goes to zero. But these flow equations suffer from the same problems and it is not clear if the resulting effective action is free of these inconsistencies.

We propose here another way of computing the effective action of a given bare theory, by introducing a control parameter λ in the theory and looking at the evolution of the effective action (defined, as usual, by the Legendre transform of the generator functional for the connected graphs) in this parameter. The parameter in question is chosen in such a manner that for its large values the fluctuations are suppressed and for small values the original bare theory is recovered. It is obvious from such a general setting that the usual "exact" evolution equations can be recovered and the former represents a certain generalization of the latter.

Interesting generalizations are the choices (i) $\lambda = M^2$, of the mass, or (ii) $\lambda = \hbar^{-1}$, the inverse of the Planck constant as the control parameter. The case (i) yields the functional generalization of the Callan-Symanzik equation. The choice (ii) produces a manifestly gauge invariant evolution equation for gauge models and represents a "renormalization group improved" loop-expansion scheme. Note that in both cases λ controls the amplitude of the fluctuations and the "evolution" in λ is the resummation of the effects of fluctuations with growing amplitude.

An essential difference between these schemes and the "exact" renormalization group procedure is that the control parameter of the latter performs the role of the cutoff. In fact, the influence of λ on the dynamics is momentum independent in the case (i) and has a weak momentum dependence in (ii) thereby a UV cutoff is required in the model. Since the evolution equation is obtained by means of the regulated, well defined path integral the scheme is genuinely non-perturbative, as for lattice regularization. Naturally the question of the convergence when one attempts to remove the cutoff after having integrated the evolution equation is rather involved and we cen offer no new results compared to the other regularization schemes.

Different standard methods based on the multiplicative renormalization scheme are briefly discussed in Section II. Our evolution equation of the effective action in control parameter which corresponds to a quadratic term in the bare action is introduced in Section III. Section IV contains the description of case (i) above, the application of our strategy to obtain the functional generalization of the Callan-Symanzik equation. The possibility of

resumming the loop expansion by solving the evolution equation, case (ii) mentioned above, is shown in Section V. We present the procedure for a scalar model only but it is obvious from the construction that, though technically more involved due to the index structure, the generalization of this scheme is trivial for gauge models. The Section VI is for the summary. The appendices give details about the Legendre transformation and the gradient expansion.

II. RENORMALIZATION GROUP SCHEMES

The traditional field theoretical methods for the renormalization group equation are based on the simplification offered by placing the ultraviolet cutoff far away from the scale of the observables. Such a separation of the scales removes the non-universal pieces of the renormalized action and the rather complicated blocking step can be simplified by retaining the renormalizable coupling constants only. The underlying formalism is the renormalized perturbation expansion, in particular the multiplicative renormalization scheme. The usual perturbative proof of the renormalizability asserts that the renormalized field and the Green functions can be written in terms of the bare quantities as

$$G_n(p_1,\cdots,p_n;g_R(\mu))_R = Z^{-\frac{n}{2}}\left(g_R,g_B,\frac{\Lambda}{\mu}\right)G_n(p_1,\cdots,p_n;g_B,\Lambda)_B\left(1+O\left(\frac{p^2}{\Lambda^2}\right)\right), \quad (1)$$

where Λ is the cutoff and the renormalized coupling constants are defined by some renormalization conditions imposed at $p^2 = \mu^2$. The evolution equation for the bare and the renormalized coupling constants result from the requirements

$$\frac{d}{d\Lambda}G_R = \frac{d}{d\Lambda}Z^{-\frac{n}{2}}G_B = 0, \quad (2)$$

$$\frac{d}{d\mu}G_B = \frac{d}{d\mu}Z^{\frac{n}{2}}G_R = 0. \quad (3)$$

Note that the non-renormalizable operators can not be treated in this fashion because the $O(p^2/\Lambda^2)$ contributions are neglected in (1). The renormalization of composite operators and the corresponding operator mixing requires the introduction of additional terms in the lagrangian. Another aspect of this shortcoming is that these methods are useful for the study of the ultraviolet scaling laws only. The study of the infrared scaling or models where there are several non-trivial scaling regimes [10] require the more powerful functional form, introduced below.

Another conventional procedure is the Callan-Symanzik equation which is based on the change of the bare mass parameter,

$$\frac{d}{dm^2}G_B = \frac{d}{dm^2}Z^{\frac{n}{2}}G_R = Z^{\frac{n}{2}}Z_{\phi^2}G_R^{comp} \quad (4)$$

where Z_{ϕ^2} is the renormalization constant for the composite operator $\phi^2(x)$ and G^{comp} is the Green function with an additional insertion of $\phi^2(p=0)$. One can convert the mass dependence inferred from the Callan-Symanzik equation into the momentum dependence by

means of dimensional analysis and the resulting expression is usually called a renormalization group equation.

The functional generalizations of the renormalization group method which are based on the infinitesimally small change of the cutoff allows up to follow the mixing of non-renormalizable operators, as well, and to trace the evolution close to the cutoff. Another advantage of these methods is that the renormalization group equation is either exact or holds in every order of the loop expansion. In the framework of the blocking transformations [1], the original work by Wegner and Houghton [2] describes the evolution of the effective action with the scale k obtained by eliminating the degrees of freedom with momenta $|p| > k$. Starting from a bare theory with cutoff $k = \Lambda$, the successive lowering of the cutoff $k \to k - \Delta k$ gives the evolution equation in dimension d

$$S_{k-\delta k}[\phi] = S_k[\phi] + \frac{1}{2}\mathrm{tr}\ln\left(\frac{\delta^2 S_k}{\delta\phi(p)\delta\phi(p)}\right) + \mathcal{O}\left(\frac{\Delta k}{k}\right)^2 \tag{5}$$

where the trace is to be take over momenta $p \in [k - \Delta k, k]$ and therefore is of order $\Delta k/k$. If we make the local potential approximation as a functional ansatz for the running action, i.e. we take for any k

$$S_k[\phi] = \int dx \left[\frac{1}{2}\partial_\mu\phi_x\partial_\mu\phi_x + U_k(\phi)\right] \tag{6}$$

we then obtain for the O(N) model in the limit $\Delta k/k \to 0$ the partial differential equation

$$k\partial_k U_k(\rho) = -\frac{k^d\Omega_d}{2(2\pi)^d}\ln\left[\left(\frac{k^2 + \partial_\rho^2 U_k(\rho)}{k^2 + \partial_\rho^2 U_k(\rho_0)}\right)\left(\frac{k^2 + \frac{1}{\rho}\partial_\rho U_k(\rho)}{k^2 + \frac{1}{\rho_0}\partial_\rho U_k(\rho_0)}\right)^{N-1}\right] \tag{7}$$

where ρ is the modulus of the N-component field and Ω_d the solid angle in dimension d.

We note the existence of other works based on a smooth cutoff procedure [3]- [7] but introducing the mass scale in a similar manner.

The method we will present in this paper proposes another approach of the evolution of the effective action with a mass scale. The idea will be to develop a functional extension of the Callan-Symanzik procedure. Starting from our equations, we will make the analogy with the exact Wilsonian renormalization group and also we will find the usual well-known one loop effective action. The functional equations we obtain are exact and can be used as an alternative method to compute the effective action.

III. EVOLUTION EQUATION

Our goal is to obtain the effective action $\Gamma[\phi]$ of the Euclidean model defined by the action $S_B[\phi]$. The usual Legendre transformation yields

$$e^{W[j]} = \int D[\phi]e^{-S_B[\phi]+\int_x j_x\phi_x} \tag{8}$$

and

$$W[j] + \Gamma[\phi] = \int_x j_x \phi_x = j \cdot \phi, \tag{9}$$

the source j is supposed to be expressed in terms of

$$\phi_x = \frac{\delta W[j]}{\delta j_x}. \tag{10}$$

A cutoff Λ is assumed implicitly in the path integral and $S_B[\phi]$ stands for the bare, cutoff action.

We modify the bare action

$$S_B[\phi] \longrightarrow S_\lambda[\phi] = \lambda S_s[\phi] + S_B[\phi] \tag{11}$$

in such a manner that the model with $\lambda \to \infty$ be soluble, because the path integral for $\lambda = \infty$ contains no fluctuations. The role of the new piece in the action is to suppress the fluctuations around the minimum ϕ_∞ of the action for large value of λ and render the model perturbative.

We plan to follow the λ dependence of the effective action by integrating out the functional differential equation

$$\partial_\lambda \Gamma = \mathcal{F}_\lambda[\Gamma] \tag{12}$$

from the initial condition

$$\Gamma_{\lambda_{init}}[\phi] = \lambda_{init} S_s[\phi] + S_B[\phi], \tag{13}$$

imposed at $\lambda_{init} \approx \infty$ to $\lambda = 0$. (12) can be interpreted as a generalization of the Callan-Symanzik equation because both generate a one-parameter family of different theories organized according to the strength of the quantum fluctuations [1]. So long as the parameter λ introduces a renormalization scale, $\mu(\lambda)$, the trajectory $\Gamma_{\lambda(\mu)}[\phi]$ in the effective action space can be thought as a renormalized trajectory. Another way to interpret (12) is to consider its integration as a method which builds up the fluctuations of the model with $\lambda = \infty$ by summing up the effects of increasing the fluctuation strength infinitesimally, $\lambda \to \lambda - \Delta\lambda$. The gradient expansion is compatible with (12) if the suppression is sufficiently smooth in the momentum space, i.e. S_s is a local functional.

The starting point to find $\mathcal{F}_\lambda[\Gamma]$ is the relation

$$\partial_\lambda \Gamma[\phi] = -\partial_\lambda W[j] - \frac{\delta W[j]}{\delta j}.\partial_\lambda j + \partial_\lambda j.\phi = -\partial_\lambda W[j], \tag{14}$$

λ and ϕ being the independent variables. This relation will be used together with

$$\partial_\lambda W[j] = -e^{-W[j]} \int D[\phi] S_s[\phi] e^{-\lambda S_s[\phi] - S_B[\phi]} = -e^{-W[j]} S_s \left[\frac{\delta}{\delta j}\right] e^{W[j]}. \tag{15}$$

[1]Note that the inverse mass is proportional to the amplitude of the fluctuations.

It is useful to perform the replacement

$$\Gamma[\phi] \longrightarrow \lambda S_s[\phi] + \Gamma[\phi] \tag{16}$$

which results the evolution equation

$$\partial_\lambda \Gamma[\phi] = e^{-W[j]} S_s \left[\frac{\delta}{\delta j}\right] e^{W[j]} - S_s[\phi]. \tag{17}$$

The next question is the choice of the fluctuation-suppressing action $S_s[\phi]$. The simplest is to use a quadratic suppression term,

$$S_s[\phi] = \frac{1}{2}\int_{x,y} \phi_x \mathcal{M}_{x,y} \phi_y = \frac{1}{2}\phi \cdot \mathcal{M} \cdot \phi. \tag{18}$$

We discuss in section V the choice $S_s = S_B$ and look at the evolution of the effective action with $\hbar$. The evolution equation (17) there sums up the loop expansion and produces the dependence in $\hbar$. This particular choice will be motivated by the extension of the present work to gauge theories.

We return now to the case of a simple scalar field without local symmetry, (18). The corresponding evolution equation can be ontained from (17), and considering the relation (A4) between the functional derivatives of $W[j]$ and $\Gamma[\phi]$,

$$\begin{aligned} \partial_\lambda \Gamma[\phi] &= \frac{1}{2}\int_{x,y} \mathcal{M}_{x,y} \left[W^{(2)}_{x,y} + \phi_x \phi_y\right] - \frac{1}{2}\int_{x,y} \phi_x \mathcal{M}_{x,y} \phi_y \\ &= \frac{1}{2}\int_{x,y} \mathcal{M}_{x,y} \left[\Gamma^{(2)}_{x,y} + \lambda \mathcal{M}_{x,y}\right]^{-1} \end{aligned} \tag{19}$$

where the functional derivatives are denoted by

$$\Gamma^{(n)}_{x_1,\cdots,x_n} = \frac{\delta^n \Gamma[\phi]}{\delta\phi_{x_1}\cdots\delta\phi_{x_n}}. \tag{20}$$

(19) reads in an operator notation

$$\partial_\lambda \Gamma[\phi] = \frac{1}{2}\mathrm{Tr}\left\{\mathcal{M} \cdot \left[\lambda \mathcal{M} + \Gamma^{(2)}\right]^{-1}\right\}, \tag{21}$$

We should bear in mind that $\Gamma^{(n)}_{x_1,\cdots,x_n}$ remains a functional of the field ϕ_x.

It is illuminating to compare this result with the evolution equations presented in refs. [3]- [7] in the framework of Wilsonian renormalization group equations

$$\partial_k \Gamma[\phi] = \frac{1}{2}\mathrm{Tr}\left\{\partial_k G_k^{-1} \cdot \left[G_k^{-1} + \Gamma^{(2)}\right]^{-1}\right\}. \tag{22}$$

where the role of $S_s^{(2)}$ is played here by the propagator $G_k(p)$ which contains the scale parameter k such that the fluctuations with momenta $|p| > k$ are suppressed. The formal similarity with (19) reflects that the different schemes agree in "turning on" the fluctuations in infinitesimal steps. We will come back to this remark in section IV.

The evolution equation can be converted into a more treatable form by the means of the gradient expansion,

$$\Gamma[\phi] = \int_x \left\{ \frac{1}{2} Z_x (\partial_\mu \phi_x)^2 + U_x + O(\partial^4) \right\} \tag{23}$$

where the notation $f_x = f(\phi_x)$ was introduced. This ansatz gives

$$\begin{aligned}
\Gamma^{(1)}_{x_1} &= -\frac{1}{2} Z^{(1)} x_1 (\partial_\mu \phi_{x_1})^2 - Z_{x_1} \Box \phi_{x_1} + U^{(1)}_{x_1} \\
\Gamma^{(2)}_{x_1,x_2} &= -\frac{1}{2} \delta_{x_1,x_2} Z^{(2)}_{x_1} (\partial_\mu \phi_{x_1})^2 - \partial_\mu \delta_{x_1,x_2} Z^{(1)}_{x_1} \partial_\mu \phi_{x_1} \\
&\quad - \delta_{x_1,x_2} Z^{(1)}_{x_1} \Box \phi_{x_1} - \Box \delta_{x_1,x_2} Z_{x_1} + U^{(2)}_{x_1}
\end{aligned} \tag{24}$$

where the $f^{(n)}(\phi) = \partial^n_\phi f(\phi)$. Such an expansion is unsuitable for $W[j]$ due to the strong non-locality of the propagator but might be more successful for the effective action where the one-particle irreducible structure and the removal of the propagator at the external legs of the contributing diagrams strongly reduce the non-local effects. The replacement of this ansatz into (17) gives (c.f. Appendix B.)

$$\begin{aligned}
\partial_\lambda U_\lambda(\phi) &= \frac{1}{2} \int_p \frac{\mathcal{M}(p)}{\lambda \mathcal{M}(p) + Z_\lambda(\phi) p^2 + U^{(2)}_\lambda(\phi)} \\
\partial_\lambda Z_\lambda(\phi) &= \frac{1}{2} \int_p \mathcal{M}(p) \Bigg[- \frac{Z^{(2)}_\lambda(\phi)}{\left(\lambda \mathcal{M}(p) + Z_\lambda(\phi) p^2 + U^{(2)}_\lambda(\phi)\right)^2} \\
&\quad + 2 Z^{(1)}_\lambda(\phi) \frac{2\left(Z^{(1)}_\lambda(\phi) p^2 + U^{(3)}_\lambda(\phi)\right) + Z^{(1)}_\lambda(\phi) p^2/d}{\left(\lambda \mathcal{M}(p) + Z_\lambda(\phi) p^2 + U^{(2)}_\lambda(\phi)\right)^3} \\
&\quad - \frac{\left(Z^{(1)}_\lambda(\phi) p^2 + U^{(3)}_\lambda(\phi)\right)^2 (\lambda \Box \mathcal{M}(p) + 2 Z_\lambda(\phi))}{\left(\lambda \mathcal{M}(p) + Z_\lambda(\phi) p^2 + U^{(2)}_\lambda(\phi)\right)^4} \\
&\quad - \frac{4}{d} Z^{(1)}_\lambda(\phi) \left(Z^{(1)}_\lambda(\phi) p^2 + U^{(3)}_\lambda(\phi)\right) \frac{(\lambda p_\mu \partial_\mu \mathcal{M}(p) + 2 Z_\lambda(\phi) p^2)}{\left(\lambda \mathcal{M}(p) + Z_\lambda(\phi) p^2 + U^{(2)}_\lambda(\phi)\right)^4} \\
&\quad + \frac{2}{d} \frac{\left(Z^{(1)}_\lambda(\phi) p^2 + U^{(3)}_\lambda(\phi)\right)^2 (\lambda \partial_\mu \mathcal{M}(p) + 2 Z_\lambda(\phi) p_\mu)^2}{\left(\lambda \mathcal{M}(p) + Z_\lambda(\phi) p^2 + U^{(2)}_\lambda(\phi)\right)^5} \Bigg]
\end{aligned} \tag{25}$$

where $\int_p = \int \frac{d^d p}{(2\pi)^d}$ and we assumed that $\partial_\mu \mathcal{M}(p)$ is proportional to p_μ.

IV. MASS DEPENDENCE

We take $\lambda = m^2$ with

$$\mathcal{M}_{x,y} = \delta_{x,y} \tag{26}$$

which minimizes strength of the higher order derivative terms generated during the evolution by being a momentum independent suppression mechanism. The evolution equation is the functional differential renormalization group version of the Callan-Symanzik equation,

$$\partial_{m^2}\Gamma[\phi] = \frac{1}{2}\mathrm{Tr}\left[m^2\delta_{x,y} + \Gamma^{(2)}_{x,y}\right]^{-1}. \tag{27}$$

The projection of this functional equation onto the gradient expansion ansatz gives

$$\begin{aligned}
\partial_{m^2}U(\phi) &= \frac{1}{2}\int_p \frac{1}{Z(\phi)p^2 + m^2 + U^{(2)}(\phi)} \\
\partial_{m^2}Z(\phi) &= \frac{1}{2}\int_p \Bigg[-\frac{Z^{(2)}(\phi)}{\left(Z(\phi)p^2 + m^2 + U^{(2)}(\phi)\right)^2} \\
&\quad + 2Z^{(1)}(\phi)\frac{p^2/dZ^{(1)}(\phi) + 2\left(Z^{(1)}(\phi)p^2 + U^{(3)}(\phi)\right)}{\left(Z(\phi)p^2 + m^2 + U^{(2)}(\phi)\right)^3} \\
&\quad - 2Z(\phi)\frac{\left(Z^{(1)}(\phi)p^2 + U^{(3)}(\phi)\right)^2}{\left(Z(\phi)p^2 + m^2 + U^{(2)}(\phi)\right)^4} \\
&\quad - \frac{8p^2}{d}Z(\phi)Z^{(1)}(\phi)\frac{\left(Z^{(1)}(\phi)p^2 + U^{(3)}(\phi)\right)}{\left(Z(\phi)p^2 + m^2 + U^{(2)}(\phi)\right)^4} \\
&\quad + \frac{8p^2}{d}Z^2(\phi)\frac{\left(Z^{(1)}(\phi)p^2 + U^{(3)}(\phi)\right)^2}{\left(Z(\phi)p^2 + m^2 + U^{(2)}(\phi)\right)^5}\Bigg]
\end{aligned} \tag{28}$$

It is important to bear in mind that we are dealing here with a well regulated theory and that the procedure described here does not aim at removing the cutoff Λ which remains an essential parameter.

Let us now simplify the differential equation for $U(\phi)$ and $Z(\phi)$ by integrating over p in (28) with sharp momentum cutoff Λ^2 in four dimensions,

$$\begin{aligned}
\partial_{m^2}U(\phi) &= \frac{1}{32\pi^2 Z(\phi)}\left[\Lambda^2 - \frac{m^2 + U^{(2)}(\phi)}{Z(\phi)}\ln\left(1 + \frac{Z(\phi)\Lambda^2}{m^2 + U^{(2)}(\phi)}\right)\right] \\
\partial_{m^2}Z(\phi) &= \frac{1}{32\pi^2 Z(\phi)}\Bigg[\frac{1}{Z^2(\phi)}\left(\frac{5}{2}\left(Z^{(1)}(\phi)\right)^2 - Z(\phi)Z^{(2)}(\phi)\right)\ln\left(1 + \frac{Z(\phi)\Lambda^2}{m^2 + U^{(2)}(\phi)}\right) \\
&\quad + \frac{1}{Z^2(\phi)}\left(Z(\phi)Z^{(2)}(\phi) - \frac{43}{12}\left(Z^{(1)}(\phi)\right)^2\right) \\
&\quad + \frac{1}{Z(\phi)}\frac{Z^{(1)}(\phi)U^{(3)}(\phi)}{\left(m^2 + U^{(2)}(\phi)\right)} - \frac{1}{6}\frac{\left(U^{(3)}(\phi)\right)^2}{\left(m^2 + U^{(2)}(\phi)\right)^2}\Bigg]
\end{aligned} \tag{29}$$

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[2] The sharp momentum space cutoff generates nonlocal interactions. Since these nonlocal contributions come from the surface terms of the loop integrals they are suppressed in a renormalizable theory when Λ is kept large. Thus the gradient expansion ansatz can be justified for the evolution (28).

In the local potential approximation $Z = 1$ we obtain

$$\partial_{m^2} U(\phi) = -\frac{m^2 + U^{(2)}(\phi)}{32\pi^2} \ln\left(1 + \frac{\Lambda^2}{m^2 + U^{(2)}(\phi)}\right) \tag{30}$$

after removing a field independent term. In order to simplify the scaling relations we consider the regime $m^2 \gg U^{(2)}$, where

$$\partial_{m^2} U(\phi) = -\frac{1}{32\pi^2} \ln\left(\frac{m^2 + \Lambda^2}{m^2}\right) U^{(2)}(\phi) \tag{31}$$

Let us come back to the analogy with the infinitesimal Wilsonian renormalization group method. In the case of a sharp cutoff k, the evolution equation in the local potential approximation in dimension $d = 4$ for $N = 1$ is given by (7):

$$k\partial_k U(\phi) = -\frac{k^4}{16\pi^2} \ln\left(\frac{k^2 + U^{(2)}(\phi)}{k^2 + U^{(2)}(\phi_0)}\right) \tag{32}$$

which reads in the same regime $k^2 \gg U^{(2)}$

$$k\partial_k U(\phi) = -\frac{k^2}{16\pi^2} U^{(2)}(\phi). \tag{33}$$

after removing a field independent term. The evolutions (31) and (33) agree if we impose the differential relation between the scale and mass parameters k and m

$$2k\frac{dk}{dm^2} = \ln\left(\frac{m^2 + \Lambda^2}{m^2}\right) \tag{34}$$

We obtain in this manner the usual justification of calling the Callan-Symanzik equation a renormalization group method where the mass scale m plays the role of a running cutoff k. The equivalence of the scales and the elimination of the non-universal contributions requires that the cutoff should be far above the mass, $m^2 \ll \Lambda^2$.

The non-vanishing anomalous dimension can be recovered with (29) In fact, when $Z \neq 1$ the relation (34) becomes field dependent according to the first equation of (29). It is worthwhile comparing what (29) gives in the asymptotical regime $m^2 \gg U^{(2)}$,

$$\partial_{m^2} Z_{m^2}(\phi) = -\frac{1}{32\pi^2 Z^3_{m^2}(\phi)} \ln\left(\frac{Z_{m^2}(\phi)\Lambda^2 + m^2}{m^2}\right) \left[Z_{m^2}(\phi) Z^{(2)}_{m^2}(\phi) - \frac{5}{2}\left(Z^{(1)}_{m^2}(\phi)\right)^2\right] \tag{35}$$

with the prediction of the Wegner-Houghton equation. A possible attempt to save the gradient expansion with sharp cutoff for the latter is the following: The contributions to the coefficient functions of the gradient, such as $Z_k(\phi)$, come from taking the derivative of the loop integral, the trace in (5), with respect to the momentum of the infrared background field $\tilde{\phi}(x)$. There are two kind of contributions, one which comes form the derivative of the integrand, another from the external momentum dependence of the limit of the integration. It is easy to verify that the ϵ-dependent non-local contributions come form the second types only [5]. Thus one may consider the approximation where these contributions are simply

neglected, assuming a cancellation mechanism between the successive blocking steps. The result is, for $k^2 \gg U_k^{(2)}(\phi)$, c.f. Appendix C,

$$k\partial_k Z_k(\phi) = -\frac{k^2}{32\pi^2 Z_k^2(\phi)}\left[2Z_k(\phi)Z_k^{(2)}(\phi) - \frac{5}{2}\left(Z_k^{(1)}(\phi)\right)^2\right]. \tag{36}$$

The formal similarity between the two different schemes, (35) and (36), can be considered as a measure of the cancellation of the non-local terms evoked above.

Finally, we show that we recover the well-known one loop effective action. For this we consider the solution of (29) in the independent mode approximation where the m^2 dependence is ignored in the integrals, $U(\phi) = U_B(\phi)$ and $Z(\phi) = 1$. We get

$$\begin{aligned} U_{eff}(\phi) &= U_B(\phi) + \frac{1}{2}\int_{M^2}^0 dm^2 \int_p \frac{1}{p^2 + m^2 + U_B^{(2)}(\phi)} \\ &= U_B(\phi) + \frac{1}{2}\int_p \ln[p^2 + U_B^{(2)}(\phi)] + O(M^{-2}), \end{aligned} \tag{37}$$

which reproduces the usual one-loop effective potential for $M \gg \Lambda$. For the kinetic term, the integration of (29) in the same approximation leads to

$$\begin{aligned} Z_{eff}(\phi) &= 1 - \frac{1}{192\pi^2}\int_{M^2}^0 dm^2 \frac{\left(U_B^{(3)}(\phi)\right)^2}{\left(m^2 + U_B^{(2)}(\phi)\right)^2} \\ &= 1 + \frac{1}{192\pi^2}\frac{\left(U_B^{(3)}(\phi)\right)^2}{U_B^{(2)}(\phi)} + O(M^{-2}), \end{aligned} \tag{38}$$

for $d = 4$ which reproduces the one-loop solution found in [14]. The agreement between the independent mode approximation to our method and the one-loop solution is expected because the right hand side of (19) is $\mathcal{O}(\hbar)$. This can be understood as a scheme independence of the one loop gamma functions. But this agreement does not hold beyond $\mathcal{O}(\hbar)$ as indicated by the imcompatibility of (35) and (36).

V. $\hbar$ DEPENDENCE

It may happen that the quadratic suppression is not well suited to a problem. In the case $S_B[\phi]$ possesses local symmetries which should be preserved then another choice is more appropriate. The application of our procedure for a gauge model can for example be based on the choice

$$\begin{aligned} S_B[A] &= -\frac{1}{4g_B^2}\int dx F^a_{\mu\nu}F^{\mu\nu a} + S_{gf}[A], \\ S_s[A] &= -\frac{1}{4g_B^2}\int dx F^a_{\mu\nu}F^{\mu\nu a}, \end{aligned} \tag{39}$$

where S_{gf} contains the gauge fixing terms and on the application of a gauge invariant regularization scheme. As mentioned after eq. (28) we need a regulator to start with in

order to follow the dependence on the amplitude of the fluctuations. One may use lattice, analytic (asymptotically free models) or Pauli-Villars (QED) regulator to render (17) well defined. The explicit gauge invariance of $S_s[A]$ which was achieved by suppressing the gauge covariant field strength instead of the gauge field itself makes obvious the independence of the resulting flow for the gauge invariant part of the action from the choice of the gauge, S_{gf}.

We leave the issue of the gauge models for future works and we return now to the scalar theory and present the evolution equation for the ϕ^4 model with quartic suppression,

$$S_B[\phi] = S_s[\phi] = \int_x \left[\frac{1}{2}(\partial_\mu \phi_x)^2 + \frac{g_2}{2}\phi_x^2 + \frac{g_3}{3!}\phi_x^3 + \frac{g_4}{4!}\phi_x^4\right]. \tag{40}$$

The similarity of this scheme with the loop expansion suggests the replacement

$$\frac{1}{\hbar} = 1 + \lambda = 1 + \frac{1}{g}, \tag{41}$$

which yields the evolution equation

$$\partial_g \Gamma[\phi] = -\frac{1}{g^2} e^{-W[j]} S_s\left[\frac{\delta}{\delta j}\right] e^{W[j]} + \frac{1}{g^2} S_s[\phi]. \tag{42}$$

The integration of the evolution equation from $g_{in} = 0$ to $g_{fin} = \infty$ corresponds to the resummation of the loop expansion, i.e. the integration between $\hbar_{in} = 0$ and $\hbar_{fin} = 1$.

The gradient expansion ansatz (23) with $Z = 1$ gives (c.f. Appendix A)

$$\begin{aligned}
\partial_g U(\phi) = -\frac{1}{g^2}\Bigg\{ & \frac{1}{2}\int_p (p^2 + g_2) G(p) \\
& + \frac{g_3}{3!}\left[3\phi \int_p G(p) - \int_{p_1,p_2} G(p_1)G(p_2)G(-p_1-p_2)\left(U^{(3)}(\phi) + g^{-1}(g_3 + g_4\phi)\right)\right] \\
& + \frac{g_4}{4!}\Bigg[3\left(\int_p G(p)\right)^2 + 6\phi^2 \int_p G(p) \\
& \quad - \int_{p_1,p_2,p_3} G(p_1)G(p_2)G(p_3)G(-p_1-p_2-p_3)\left(U^{(4)}(\phi) + g^{-1}g_4\right) \\
& \quad - 3\int_{p_1,p_2,p_3} G(p_1)G(p_2)G(p_3)G(-p_1-p_2)G(-p_1-p_2-p_3) \\
& \qquad \times \left(U^{(3)}(\phi) + g^{-1}(g_3 + g_4\phi)\right)^2 \\
& \quad - 4\phi \int_{p_1,p_2} G(p_1)G(p_2)G(-p_1-p_2)\left(U^{(3)}(\phi) + g^{-1}(g_3 + g_4\phi)\right)\Bigg]\Bigg\},
\end{aligned} \tag{43}$$

where we used the fact that the Fourier transform of the 1PI amplitude for $n \geq 3$ and $Z = 1$ is

$$\int_{x_1,\cdots,x_n} e^{i(p_1\cdot x_1 + \cdots + p_n \cdot x_n)} \Gamma^{(n)}(x_1,\cdots,x_n) = (2\pi)^d \delta(p_1 + \cdots + p_n) U^{(n)}(\phi). \tag{44}$$

The propagator in the presence of the homogeneous background field ϕ is given by

$$G(p) = \left[p^2 + U^{(2)}(\phi) + g^{-1}\left(p^2 + g_2 + g_3\phi + \frac{g_4}{2}\phi^2\right)\right]^{-1}. \tag{45}$$

Since the momentum dependence in the right hand side of (43) is explicit and simple the one, two and three loop integrals can be carried out easily by means of the standard methods. The successive derivatives of the resulting expression with respect to ϕ yield the renormalization group coefficient functions.

The use of our functional equations described in this section shows that this method can be generalized to any kind of action S_s and not only to a quadratic suppression term, as shown in the previous sections.

VI. SUMMARY

The formal strategy of the renormalization group is generalized in this paper in such a manner that it includes the Callen-Symanzik scheme and the resummation of the loop-expansion as two possibilities. These kinds of generalization depart from the original spirit of the renormalization group program because the resulting flow does not correspond to the same physics, instead it interpolates between a suitable chosen, perturbatively solvable initial condition and the actual bare model. This property is shared with usual the "exact" renormalization group schemes where the dependence of the effective action on an IR cutoff is followed.

Our scheme can be considered as a renormalization group method in the space of the field amplitudes what is usually called the internal space. The evolution in the control parameter along the flow corresponds to the taking into account the contributions of fluctuations with increasing strength. Such an iterative inclusion of the fluctuations according to their scale parameter in the space of the amplitudes, instead of their scale parameter in the space-time, their momentum, is the difference between the usual renormalization group procedure and the ones described in this article.

Note added in proof: After this work has been completed we learned that a method presented for gauge models in ref. [15] is similar to ours in the case of mass dependence (section IV). [15] gives a loop expanded solution of the exact equation, whereas our solution is built in the framework of the derivative expansion. Finally, our approach can be generalized to any kind of suppression action S_s which is compatible with the symmetries as shown in section V.

APPENDIX A: LEGENDRE TRANSFORMATION

We collect in this Appendix the relations between the derivatives of the generator functional $W[j]$ and $\Gamma[\phi]$ used in obtaining the evolution equations for Γ.

We start with the definitions

$$W[j] + \Gamma[\phi] + \lambda S_s[\phi] = j \cdot \phi, \tag{A1}$$

and

$$\phi_x = W_x^{(1)}. \tag{A2}$$

The first derivative of Γ gives the inversion of (A2),

$$\Gamma^{(1)}_{x} = j_x - \lambda S^{(1)}_{s,x}. \tag{A3}$$

The second derivative is related to the propagator $W^{(2)}_{x_1,x_2} = G_{x_1,x_2}$

$$\Gamma^{(2)}_{x_1,x_2} = \frac{\delta j_{x_1}}{\delta \phi_{x_2}} - \lambda S^{(2)}_{s,x_1,x_2} = G^{-1}_{x_1,x_2} - \lambda S^{(2)}_{s,x_1,x_2}. \tag{A4}$$

The third derivative is obtained by differentiating (A4),

$$\Gamma^{(3)}_{x_1,x_2,x_3} = -\int_{y_1,y_2,y_3} G^{-1}_{x_1,y_1} G^{-1}_{x_2,y_2} G^{-1}_{x_3,y_3} W^{(3)}_{y_1,y_2,y_3} - \lambda S^{(3)}_{s,x_1,x_2,x_3}. \tag{A5}$$

The inverted form of this equation is

$$W^{(3)}_{x_1,x_2,x_3} = -\int_{y_1,y_2,y_3} G_{x_1,y_1} G_{x_2,y_2} G_{x_3,y_3} \left(\Gamma^{(3)}_{y_1,y_2,y_3} + \lambda S^{(3)}_{s,y_1,y_2,y_3}\right). \tag{A6}$$

The further derivation gives

$$\begin{aligned}\Gamma^{(4)}_{x_1,x_2,x_3,x_4} = \int_{y_1,y_2,y_3,y_4,z_1,z_2} \Big[& G^{-1}_{x_1,y_1} G^{-1}_{x_2,y_2} G^{-1}_{x_3,y_3} G^{-1}_{x_4,y_4} W^{(4)}_{y_1,y_2,y_3,y_4} \\ & + G^{-1}_{x_1,y_1} G^{-1}_{x_2,y_2} W^{(3)}_{y_1,y_2,z_1} G^{-1}_{z_1,z_2} W^{(3)}_{z_2,y_3,y_4} G^{-1}_{x_3,y_3} G^{-1}_{x_4,y_4} \\ & + G^{-1}_{x_3,y_3} G^{-1}_{x_2,y_2} W^{(3)}_{y_3,y_2,z_1} G^{-1}_{z_1,z_2} W^{(3)}_{z_2,y_1,y_4} G^{-1}_{x_1,y_1} G^{-1}_{x_4,y_4} \\ & + G^{-1}_{x_1,y_1} G^{-1}_{x_4,y_4} W^{(3)}_{y_1,y_4,z_1} G^{-1}_{z_1,z_2} W^{(3)}_{z_2,y_3,y_2} G^{-1}_{x_3,y_3} G^{-1}_{x_2,y_2} \Big] \\ & - \lambda S^{(4)}_{s,x_1,x_2,x_3,x_4}. \end{aligned} \tag{A7}$$

Its inversion expresses the four point connected Green function in terms of the 1PI amplitudes,

$$\begin{aligned} W^{(4)}_{x_1,x_2,x_3,x_4} = \int_{y_1,y_2,y_3,y_4,z_1,z_2} \Big[& G_{x_1,y_1} G_{x_2,y_2} G_{x_3,y_3} G_{x_4,y_4} \left(\Gamma^{(4)}_{y_1,y_2,y_3,y_4} + \lambda S^{(4)}_{s,y_1,y_2,y_3,y_4}\right) \\ & - G_{x_1,y_1} G_{x_2,y_2} \left(\Gamma^{(3)}_{y_1,y_2,z_1} + \lambda S^{(3)}_{s,y_1,y_2,z_1}\right) G_{z_1,z_2} \left(\Gamma^{(3)}_{z_2,y_3,y_4} + \lambda S^{(3)}_{s,z_2,y_3,y_4}\right) G_{x_3,y_3} G_{x_4,y_4} \\ & - G_{x_3,y_3} G_{x_2,y_2} \left(\Gamma^{(3)}_{y_3,y_2,z_1} + \lambda S^{(3)}_{s,y_3,y_2,z_1}\right) G_{z_1,z_2} \left(\Gamma^{(3)}_{z_2,y_1,y_4} + \lambda S^{(3)}_{s,z_2,y_1,y_4}\right) G_{x_1,y_1} G_{x_4,y_4} \\ & - G_{x_1,y_1} G_{x_4,y_4} \left(\Gamma^{(3)}_{y_1,y_4,z_1} + \lambda S^{(3)}_{s,y_1,y_4,z_1}\right) G_{z_1,z_2} \left(\Gamma^{(3)}_{z_2,y_3,y_2} + \lambda S^{(3)}_{s,z_2,y_3,y_2}\right) G_{x_3,y_3} G_{x_2,y_2} \Big]. \end{aligned} \tag{A8}$$

APPENDIX B: EVOLUTION EQUATION DERIVATION

We give here some details on the computation of (25). To get the evolution equation of the potential part of the gradient expansion (23), one has to take a homogeneous field $\phi = \phi_0$ in (19). But to distinguish the kinetic contribution from the potential one, a non homogeneous field $\phi(x) = \phi_0 + \eta(x)$ is needed, as well. Let k be the momentum where the field η is non-vanishing. Then the effective action can be written as

$$\Gamma[\phi] = V_d U_\lambda(\phi_0) + \frac{1}{2}\int_q \tilde{\eta}(q)\tilde{\eta}(-q)\left(Z_\lambda(\phi_0)q^2 + U_\lambda^{(2)}(\phi_0)\right) + \mathcal{O}(\tilde{\eta}^3, k^4) \tag{B1}$$

where V_d is the spatial volume. Thus we need the second derivative of the effective action in (19) up to the second order in $\tilde{\eta}$ to identify the different contributions. The terms independent of $\tilde{\eta}$ give the equation for U_λ and the ones proportional to $k^2\tilde{\eta}^2$ the equation for Z_λ. The contributions proportional to $\tilde{\eta}^2$ but independent of k yield an equation for $U_\lambda^{(2)}$ which must be consistent with the equation for U_λ. The result is

$$\begin{aligned}\Gamma^{(2)}_{p_1,p_2} &= \left[Z_\lambda(\phi_0)p_1^2 + U_\lambda^{(2)}(\phi_0)\right]\delta(p_1+p_2) \\ &+ \int_q \tilde{\eta}(q)\left[Z_\lambda^{(1)}(\phi_0)(p_1^2+q^2+qp_1) + U_\lambda^{(3)}(\phi_0)\right]\delta(p_1+p_2+q) \\ &+ \frac{1}{2}\int_{q_1,q_2}\tilde{\eta}(q_1)\tilde{\eta}(q_2)\left[Z_\lambda^{(2)}(\phi_0)(p_1^2+2q_1^2+q_1q_2+2q_1p_1) + U_\lambda^{(4)}(\phi_0)\right]\delta(p_1+p_2+q_1+q_2) \\ &+ O(\tilde{\eta}^3, k^4)\end{aligned} \tag{B2}$$

Finally one computes the inverse of the operator $\lambda\mathcal{M}_{p_1,p_2} + \Gamma^{(2)}_{p_1,p_2}$ and expands it in powers of $\tilde{\eta}$ and k. The trace over p_1 and p_2 needs the computations of terms like

$$\mathrm{Tr}\left\{(p_1q_1)(p_2q_2)F(p_1,p_2)\delta(p_1+p_2+q_1+q_2)\right\} = \frac{q_1^2}{d}\delta(q_1+q_2)\int_p p^2 F(p,-p) \tag{B3}$$

and they lead to (25). The consistency with the equation for $U_\lambda^{(2)}$ is satisfied.

APPENDIX C: EXACT RENORMALIZATION GROUP EQUATIONS

For the derivation of the flow equations for the potential U_k and the wave function renormalization Z_k we will take in the Wegner-Houghton equation (5) a non homogeneous background field $\phi = \phi_0 + \phi_l$ where ϕ_0 is homogeneous and $\phi_l = \sum_{|q|=l}\phi_q e^{iqx}$. The expansion of the running action Γ_k (in the sens of the renormalization group transformations) in powers of ϕ_l gives

$$\partial_k S_k[\phi_0+\phi_l] = V_d\left(\partial_k U_k(\phi_0) + \frac{\Phi^2}{2}\left[l^2\partial_k Z_k(\phi_0) + \partial_k U_k^{(2)}(\phi_0)\right] + ...\right) \tag{C1}$$

where $\Phi^2 = \sum_q \phi_q\phi_{-q}$, so that the expansion of (5) in powers of ϕ_l will help us identify:

- the evolution of U_k given by the terms independent of Φ^2,
- the evolution of Z_k given by the terms proportional to $l^2\Phi^2$,
- the evolution of $U_k^{(2)}$ given by the terms proportional to Φ

but independent of l. This equation will of course have to be consistent with the one for U_k. For this consistency condition, we will need to take $l << k$, as will be shown.

The Wegner-Houghton equation needs the second derivative of the action which reads in the second order in ϕ_l as

$$\frac{1}{V_d}\frac{\partial^2 S_k}{\partial\tilde\phi_{p_1}\partial\tilde\phi_{-p_2}}|_{\phi_0+\phi_B} = \left[p_1^2 Z_0 + U_0^{(2)}\right]\delta(p_1-p_2) \tag{C2}$$
$$+\sum_q \tilde\phi_q\left[\left(p_1^2+q^2+qp_1\right)Z_0^{(1)}+U_0^{(3)}\right]\delta(p_1-p_2+q)$$
$$+\frac{1}{2}\sum_{q_1,q_2}\tilde\phi_{q_1}\tilde\phi_{q_2}\left[\left(p_1^2+2q_1^2+q_1q_2+2q_1p_1\right)Z_0^{(2)}+U_0^{(4)}\right]\delta(p_1-p_2+q_1+q_2).$$

The expansion of the logarithm in the same order reads

$$\ln\left[\frac{1}{V_d}\frac{\partial^2 S_k}{\partial\tilde\phi_{p_1}\partial\tilde\phi_{p_2}}\right] = \delta(p_1-p_2)\ln\left[p_1^2 Z_0+U_0^{(2)}\right] \tag{C3}$$
$$+\frac{1}{p_1^2Z_0+U_0^{(2)}}\sum_q\delta(p_1-p_2+q)\tilde\phi_q\left[\left(p_1^2+q^2+qp_1\right)Z_0^{(1)}+U_0^{(3)}\right]$$
$$+\frac{1}{2}\frac{1}{p_1^2Z_0+U_0^{(2)}}\sum_{q_1,q_2}\delta(p_1-p_2+q_1+q_2)\tilde\phi_{q_1}\tilde\phi_{q_2}\Bigg[\left[\left(p_1^2+2q_1^2+q_1q_2+2q_1p_1\right)Z_0^{(2)}+U_0^{(4)}\right]$$
$$-\frac{1}{(p_2-q_2)^2Z_0+U_0^{(2)}}\left[\left(p_1^2+q_1^2+q_1p_1\right)Z_0^{(1)}+U_0^{(3)}\right]\left[\left(p_2^2+q_2^2-q_2p_2\right)Z_0^{(1)}+U_0^{(3)}\right]\mathcal{C}(p_1-q_1)\Bigg]$$

where $\mathcal{C}(p_1-q_1)$ represents the constraint that $|p_1-q_1|$, as well as $|p_1|$, must be between k and $k-\delta k$. This is the origin of the constraint $l << k$: if this is not satisfied, the term proportional to $\mathcal{C}(p_1-q_1)$ in (C3) does not contribute to the evolution equations (it is of the order δk^2) and the evolution of $U_k^{(2)}$ is not consistent with the one of U_k.

Then we need to expand (C3) in powers of l. But there appears terms proportional to the first power of l which would give non local contributions to the gradient expansion since they would be written $\sqrt{\Box}$. There are two ways to rid of the non-local contributions when the model is solved by the loop expansion, i.e. by means of loop integrals for momenta $0\le p\le\Lambda$. One is to use lattice regularization where the periodicity in the Brillouin zone cancel the q dependence of the domain of the integration. Another way to eliminate the non-local terms is to remove the cutoff. Since the non-local contributions represent surface terms they vanish as $\Lambda\to\infty$.

One may furthermore speculate that some of the non-local terms cancel between the consecutive steps of the blocking $k\to k-\Delta k$ for a suitable choice of the cutoff function $f(\kappa)$ in the propagator $G_k^{-1}(p)=f(p/k)G^{-1}(p)$. Ignoring simply the non-local terms the identification of the coefficients of the different powers in the derivative expansion, we finally obtain from the Wegner-Houghton equation

$$k\partial_k U_k(\phi_0) = -\frac{\hbar\Omega_d k^d}{2(2\pi)^d}\ln\left(\frac{Z_k(\phi_0)k^2+U_k^{(2)}(\phi_0)}{Z_k(0)k^2+U_k^{(2)}(0)}\right) \tag{C4}$$
$$k\partial_k Z_k(\phi_0) = -\frac{\hbar\Omega_d k^d}{2(2\pi)^d}\left(\frac{Z_k^{(2)}(\phi_0)}{Z_k(\phi_0)k^2+U_k^{(2)}(\phi_0)}-2Z_k^{(1)}(\phi_0)\frac{Z_k^{(1)}(\phi_0)k^2+U_k^{(3)}(\phi_0)}{\left(Z_k(\phi_0)k^2+U_k^{(2)}(\phi_0)\right)^2}\right.$$
$$\left.-\frac{k^2}{d}\frac{\left(Z_k^{(1)}(\phi_0)\right)^2}{\left(Z_k(\phi_0)k^2+U_k^{(2)}(\phi_0)\right)^2}+\frac{4k^2}{d}Z_k(\phi_0)Z_k^{(1)}(\phi_0)\frac{Z_k^{(1)}(\phi_0)k^2+U_k^{(3)}(\phi_0)}{\left(Z_k(\phi_0)k^2+U_k^{(2)}(\phi_0)\right)^3}\right.$$

$$+ \; Z_k(\phi_0)\frac{\left(Z_k^{(1)}(\phi_0)k^2 + U_k^{(3)}(\phi_0)\right)^2}{\left(Z_k(\phi_0)k^2 + U_k^{(2)}(\phi_0)\right)^3} - \frac{4k^2}{d} Z_k^2(\phi_0)\frac{\left(Z_k^{(1)}(\phi_0)k^2 + U_k^{(3)}(\phi_0)\right)^2}{\left(Z_k(\phi_0)k^2 + U_k^{(2)}(\phi_0)\right)^4}\Bigg)$$

where the origin of the potential has been chosen at $\phi_0 = 0$.

When $k^2 \gg U_k^{(2)}(\phi)$ this gives (36) in dimension $d = 4$.

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